Chapter 5 — Exercises (Haar / Hadamard)

Gallier & Quaintance — workbook companion (core set)

Q1 (core). Using the 4×4 Haar change-of-basis idea in §5.1, compute the first averages and half-differences for the signal $v = (6, 4, 5, 1)$.
Show your working in the box below.

Working:

Q2 (core). State Proposition 5.2 (Sylvester) in your own words, then build H_4 from H_2.

Working:

Q3 (stretch). Why does setting small Haar coefficients to zero still let you reconstruct a recognisable signal/image? Answer in 3–5 sentences.

Working:

Chapter 5 — Answer sheet (worked)

Q1 — worked solution

Pair adjacent samples: (6,4) and (5,1).

Averages: (6+4)/2 = 5, (5+1)/2 = 3.

Half-differences: (6-4)/2 = 1, (5-1)/2 = 2.

So first-stage coeffs include averages (5,3) and diffs (1,2).

(Exact matrix form depends on the W basis used in §5.1; mark method for pairing + average/diff.)

Q2 — worked solution

Sylvester: if H is Hadamard n×n, then the block matrix [[H,H],[H,-H]] is Hadamard 2n×2n.

Start from H2 = [[1,1],[1,-1]].

H4 = [[H2,H2],[H2,-H2]] =

[[1,1,1,1],[1,-1,1,-1],[1,1,-1,-1],[1,-1,-1,1]].

Q3 — worked solution (stretch)

Haar concentrates energy in coarse averages; many detail coeffs are near zero for smooth signals.

Zeroing tiny coeffs removes fine noise/detail but keeps the large-scale shape (lossy compression).

Reconstruction from the remaining coeffs therefore still looks similar to the original.