

Chapter 5 — Answer sheet (worked)

Q1 — worked solution

Pair adjacent samples: (6,4) and (5,1).

Averages: $(6+4)/2 = 5$, $(5+1)/2 = 3$.

Half-differences: $(6-4)/2 = 1$, $(5-1)/2 = 2$.

So first-stage coeffs include averages (5,3) and diffs (1,2).

(Exact matrix form depends on the W basis used in §5.1; mark method for pairing + average/diff.)

Q2 — worked solution

Sylvester: if H is Hadamard $n \times n$, then the block matrix $\begin{bmatrix} H & H \\ H & -H \end{bmatrix}$ is Hadamard $2n \times 2n$.

Start from $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

$H_4 = \begin{bmatrix} H_2 & H_2 \\ H_2 & -H_2 \end{bmatrix} =$

$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$.

Q3 — worked solution (stretch)

Haar concentrates energy in coarse averages; many detail coeffs are near zero for smooth signals.

Zeroing tiny coeffs removes fine noise/detail but keeps the large-scale shape (lossy compression).

Reconstruction from the remaining coeffs therefore still looks similar to the original.